The Dominating Set Problem is Fixed Parameter Tractable for Graphs of Bounded Genus

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Abstract. We describe an algorithm for the dominating set problem with time complexity $O((24g^2 + 24g + 1)^k n^2)$ for graphs of bounded genus g, where k is the size of the set. It has previously been shown that this problem is fixed parameter tractable for planar graphs. Our method is a refinement of the earlier techniques.

Keywords. graph, genus, dominating set, fixed parameter algorithm.

1 Introduction

The DOMINATING SET problem is defined as follows.

Input: A graph $G = (V, E)$ and an integer parameter k
Question: Does there exist a set of nodes $V' \subseteq V$ such that $|V'| \leq k$ and such that for all nodes $v \in V$ either $v \in V'$ or there is an edge $uv \in E$ and $u \in V'$.

This is a classic NP-complete problem which is also apparently not *fixed parameter tractable* because it is known to be $W[2]$-complete in the W-hierarchy of fixed parameter complexity theory [DF99]. In this theory, any problem for which there is an algorithm with time complexity $O(f(k)n^\alpha)$, for some problem parameter k, where n is the number of nodes in the graph and where α is a constant independent of k and n, is said to be *fixed parameter tractable.*

The dominating set problem remains NP-complete when restricted to planar graphs. However, Fellows and Downey [DF95,DF99] gave a "search tree" algorithm for this problem which has time complexity $O(11^k n)$, when the input is restricted to planar graphs. The best search tree algorithm so far, which uses *kernelization*, is described in [AFF$^+$01] and has a time complexity of $O(8^k n)$. A tree decomposition based algorithm with time complexity $O(c^{\sqrt{k}} n)$, where $c = 4^{6\sqrt{34}}$, is given in [ABFN00]. Related work includes [AFN01a,AFN01b]. Recently, in [AFN02], it has been shown that the problem can be reduced to a kernel of size $214k$.

In this paper, using the search tree approach, we show that the dominating set problem is fixed parameter tractable for graphs of bounded genus. For graphs of genus g we prove a time complexity of $O((24g^2 + 72g + 1)^k n^2)$.

2 The Algorithms

2.1 The Basic Algorithm

The result in [AFF+01] is based on the following simple algorithm, Figure 1. The node set B is a subset of the nodes incident with the edges in the edge set E and *exists* is a global Boolean variable. $N(v)$ denotes the neighbors of v (not including v itself) with respect to the edge set E and $I(v)$ denotes the set of edges incident with v. The claim that the global variable *exists* is set to true iff

procedure dominating_set (B : node_set, E : edge_set, k : integer);

{B is a subset of the nodes incident with edges in E.
The global variable *exists* is set to true iff there exists a dominating set for the nodes in B (with respect to the edges in E) of cardinality not greater than k}

if $|B| \leq k$ **then** exists $\leftarrow$ true
else if $k > 0$
  then Choose some $u \in B$;
    for all $v \in (N(u) \bigcup \{u\})$ **do**
      dominating_set $(B - (N(v) \bigcup \{v\}), E - I(v), k - 1)$;

Fig. 1. The Basic Algorithm

there exists a dominating set for the nodes in B of cardinality not greater than k follows from the observation that for every node u in B either u or one of its neighbors is in the dominating set. Hence we can decide the dominating set question for any graph $G = (V, E)$ and integer k by setting *exists* to false and invoking dominating_set(V, E, k).

Suppose the maximum degree of any node in B is d and let $t(n)$ denote the time complexity of the procedure, excluding the recursive invocation, where $n = |E|$. Then the time complexity $T(n)$ of the procedure is described by the recurrence relation: $T(n, k) \leq t(n) + (d + 1)T(n - 1, k - 1)$. The solution to this recurrence is $T(n, k) = O((d + 1)^k t(n))$, as can be seen by substituting $T(n, k) \leq (c(d + 1)^k - b)t(n)$, for some constants c and b, in the recurrence and assuming that $d \geq 2$ as we will see is the case.

It is straightforward to adjust node and edge sets and even to copy entire data structures for the recursive call, all in time linear in the number of edges. Noting that for graphs of bounded genus the number of edges is $O(n)$, we see that this simple algorithm has time complexity $O((d+1)^k n)$ and the dominating set problem for any family of graphs with bounded degree is fixed parameter tractable.

2.2 The Reduction Procedure

To make use of this algorithm for graphs of bounded genus, but not of bounded degree, a reduction procedure is applied to the graph at each level of recursion. This procedure does not change the minimum size of a dominating set but guarantees that there is at least one node in B of bounded degree. Figure 2 defines the reduction procedure. Rules R1 through R6 are among the reduction rules used in [AFF+01]. R7 is a new rule which generalizes one of the old rules. Given this procedure, we replace the statement "Choose some $u \in B$" in the basic algorithm by two statements: "Invoke Reduce; Choose some $u \in B$ of minimum degree".

procedure Reduce;
{We assume the node sets B and E as in Figure 1.
For convenience, we refer to the nodes in B as the "black" nodes and those nodes incident with some edge in E, but not in B, as the "white" nodes.}

repeat

R1: Remove all edges between white nodes from E;

R2: Remove all edges incident with degree 1 white nodes from E;

R3: If there is a black node w of degree 1, incident with a node u,
then remove all neighbors of u from B; add u to B;
remove all edges incident with u from E;

R4: If there is a white node u of degree 2, with black neighbors u_1 and u_2,
and an edge $u_1u_2 \in E$, then remove the edges uu_1 and uu_2 from E;

R5: If there is a white node u of degree 2, with black neighbors u_1, u_3,
and there is a black node u_2 and edges u_1u_2 and $u_2u_3 \in E$,
then remove the edges uu_1 and uu_3 from E;

R6: If there is a white node u of degree 3, with black
neighbors u_1, u_2, u_3 and there are edges u_1u_2 and $u_2u_3 \in E$
then remove the edges uu_1, uu_2 and uu_3 from E;

R7: If there are white nodes u_1 and u_2 such that $d(u_1) \leq 7$ and $N(u_1) \subseteq N(u_2)$,
then remove all the edges $\in E$ incident with u_1 from E;

until no change in E;

Fig. 2. The Reduction Procedure

Let B' and E' be the node and edge sets resulting from the application of any one of the rules in the reduction procedure to the sets B and E. One can

examine each of the rules to see that there exists a dominating set for the nodes in B with respect to the edges in E of cardinality k iff there exists a dominating set for the nodes in B' with respect to the edges in E' of cardinality k.

A graph whose node set is partitioned into black and white nodes will be referred to as a *black and white* graph. The black nodes are those yet to be dominated. The white nodes are those already dominated. A graph that is derived by applying the reduction procedure to a black and white graph will be referred to as a *reduced* black and white graph.

2.3 Time Complexity of the Reduction Process

We justify an upper bound of $O(n^2)$ on the the time complexity, $t(n)$, of the reduction procedure. We assume the use of two data structures representing the graph and associated information:

- An elaborated adjacency list in which each list is a two way linked list, and for all edges $\{u, v\}$ the entry for u in the list for v is linked directly to the entry for v in the list for u, and *vice versa*. With these extra links, an edge, once identified can be removed in constant time. A black/white indicator and a degree count are associated with each node.
- A standard adjacency matrix permits the existence of an edge to be tested in constant time, and likewise the removal of an edge.

With these data structures in mind, consider the work done in executing an entire sequence of k invocations of the dominating set procedure. It can be seen that for all rules except R7, $O(n)$ is an upper bound on the work involved in each rule. For rule R7, it seems that, although no work is required unless the degree of a white node is reduced to seven or less, any such node must be compared to all other white nodes. Thus we see no better bound than $O(n^2)$.

3 The Bounded Genus Case

Let $\mathcal{S}_g$ be the orientable compact surface of genus g. An embedding of a graph G in $\mathcal{S}_g$ is a drawing in $\mathcal{S}_g$ such that each node corresponds to a point and each edge to a line, i.e., a simple curve with two ends, and such that two lines corresponding to a pair of edges are internally disjoint, i.e., do not intersect or intersect only at their ends. The genus of G, written $\gamma(G)$, is the minimum g such that G can be embedded in $\mathcal{S}_g$. By an embedded graph we mean a graph together with its embedding in a surface, which allows us to talk about both combinatorial and geometrical properties. For instance, an edge of an embedded graph can also be seen as a line connecting two end points.

Let G be an embedded graph. A face, F, of G is a connected component of $\mathcal{S}_g - G$. The boundary-graph of F, written $b(F)$, is the subgraph of G induced by those nodes of G incident with F. A face is said to be a 2-cell if it is homeomorphic to an open disk. G is a 2-cell embedding if every face is a 2-cell. The boundary-graph of F could contain a cut edge, i.e., an edge whose removal disconnects

the graph. A cut edge can appear twice in a boundary. The degree of a face F, written $d_G(F)$ or $d(F)$, is the number of edges appearing in the boundary, edges appearing twice being counted twice. A face of degree d is called a d-face. If $b(F)$ is a cycle in G, then F is called a proper face. We say that a closed line in $\mathcal{S}_g$ is contractible if it is the boundary of an open disk. Two lines joining two different points are said to be transformable if they form a contractible closed line. It can be seen that, in a 2-cell embedded graph, the loop edge forming the boundary of a 1-face is contractible, and two multiple edges forming the boundary of a 2-face are transformable.

Lemma 3.1. [Tho95, Chapter 5, Theorem 3.8] *Every embedding of G in $S_\gamma(G)$ is a 2-cell embedding.*

Lemma 3.2. [Tho95, Chapter 5, Theorem 3.3] *Let $G = (V, E)$ be a connected simple graph with $|V| \geq 4$, then*

$$|E| \leq 6(\gamma(G) - 1) + 3|V| \tag{1}$$

Lemma 3.3. *For any two distinct points in $\mathcal{S}_g$ ($g \geq 1$), there are at most $6g$ internally disjoint and mutually non-transformable lines joining them.*

Proof. Suppose, on the contrary, that there are $s > 6g$ internally disjoint and mutually non-transformable lines joining two points u, v in $\mathcal{S}_g$. If we view u and v as nodes and the lines joining them as edges, together they constitute an embedded graph G with two nodes and $s > 6$ edges. G has s multiple edges, no 1-face, no 2-face and no cut edge. The following relation, for the embedding of any graph in $\mathcal{S}_g$, is proved in [Tho92]: $n - e + f \geq 2 - 2g$, where n is the number of nodes, e is the number of edges and f is the number of faces in the embedding. Here $n = 2$ and $3f \leq 2s$, because there are no 1-faces or 2-faces. Hence $s \leq 6g$, which would contradict the assumption with which we began the proof. □

Let $G = (V, E)$ be graph and x, y be nodes of G, we denote by $G + xy$ the simple graph $(V, E \cup \{xy\})$. We denote by $\delta(G)$ the minimum degree of G.

Lemma 3.4. *Let $G = (V, E)$ be a connected graph for which $\delta(G) \geq 3$. If G is not 3-connected then G has a 2-node cut $\{x, y\}$ such that there is a component C of $G - \{x, y\}$ such that $G[V(C) \cup \{x, y\}] + xy$ is 3-connected.*

Proof. Because $\delta(G) \geq 3$, $|V(G)| = 4$ implies that G is isomorphic to K_4, which is 3-connected. No smaller graph is 3-connected.

Suppose $|V(G)| > 4$ and G is not 3-connected. Then either there exists a cut node $\{x\}$ which is in a pendant block or, if not, there exists a 2-cut $\{x, y\}$.

There is no cut node

Then there exists a 2-cut $\{x, y\}$. Let C be a component of $G - \{x, y\}$ and let $G' = G[V(C) \cup \{x, y\}] + xy$. G' is 2-connected. Any such C has at least 2 nodes, else there is a node in G of degree 2. Hence all such G' contain at least 4 nodes.

If $|V'| = 4$ then $d_{G'}(x) = d_{G'}(y) = 3$, else there is a node in G of degree 2. Hence G' is isomorphic to K_4.

If $|V'| > 4$, consider the following procedure.

while G' is not 3-connected **do**
  Let $\{a, b\}$ be a 2-cut in G';
  Let C be a component in $G' - \{a, b\}$ not containing both x and y;
  $x \leftarrow a$; $y \leftarrow b$;
  $G' \leftarrow G[V(C) \cup \{x, y\}] + xy$
endwhile

G' does not contain a cut node, hence, if it is not 3-connected there is a 2-cut $\{a, b\}$ in G' such that $\{a, b\} \neq \{x, y\}$. We note that $\{a, b\}$ is also a cut in G. The procedure must terminate, with G' being 3-connected, because the number of nodes in G decreases at each iteration. If not before, it must terminate when $|V'| = 4$ because then $G' = K_4$, else one of the remaining two nodes had degree less than three in G.

There exists a cut node

There must exist a cut node x which is in some pendant block (a maximal 2-connected component of G) say B. Note that since $\delta(G) \geq 3$, B must contain at least 4 nodes.

If B is 3-connected, then $\{x, a\}$, where a is a neighbour of x in B, is a 2-cut in G satisfying the lemma.

If B is not 3-connected, then $|V(B)| > 4$ and there exists a 2-cut, say $\{u, v\}$, in B, since there are no cut nodes. One of the components, say C, in $B - \{u, v\}$ does not contain x. If $B[C \bigcup \{u, v\}] + xy$ is not three connected then, by the argument used in the 2-connected case, there exists another 2-cut in C with the desired properties.

□

Lemma 3.5. *If $G = (B \bigcup W, E)$ is reduced, then the white nodes constitute an independent set.*

Proof. This follows from rule R1 of the reduction procedure, which is inside the loop. Hence, although rule R3 can add white nodes and hence maybe edges between white nodes, rule R1 will be again enacted and remove any edges between white nodes. □

Lemma 3.6. *For all d, if there exists a reduced black and white graph G in which the minimum degree of any black node is d then there there exists a reduced graph G' in which the minimum degree of any black node is also d and no white node has degree less than 3.*

Proof. By rule R2 of the reduction procedure, G has no white nodes of degree 1. Let u be a white node of degree 2 with neighbours x and y, which are necessarily black, by rule R1. By rule R4, there is no edge xy. Let G' be obtained from G by replacing all white nodes of degree 2 by an edge between their neighbours.

Note that this operation does not change the the minimum degree of any black node. Suppose G' is not reduced. Then the only rule that could possibly apply is R6, i.e., there is some white node u of degree 3 in G' with black neighbours and edges between two pairs of these neighbours. Because G is reduced, rule R6 did not apply to u in G. Hence one of the edges between the neighbours of u must have been created in the derivation of G' from G, i.e., one of these edges replaced a white node u' of degree 2. But this implies that R7 would apply in G to u and u', contradicting the assumption that G is reduced. □

We are now ready to present the main result of this paper.

Theorem 3.7. *If $G = (B \bigcup W, E)$ is a reduced black and white graph of genus at most g, then there exists a node $b \in B$ such that $d_G(b) \leq 24g(g+3)$.*

Proof. A better bound for planar graphs, where $g = 0$, has already been proved in [AFF+01]. So let us assume that $g \geq 1$. In contradiction to the lemma, suppose that $G = (B \bigcup W, E)$ is a reduced black and white graph with genus $\gamma(G) \leq g$ and that every node $u \in B$ is such that $d_G(u) \geq 24g(g+3)+1$. We may assume that G is a connected simple graph and that, by Lemma 3.6, for any $u \in W$, $d_G(u) \geq 3$. Consider G to be embedded in $\mathcal{S}_{\gamma(G)}$. Then the embedding is a 2-cell embedding by Lemma 3.1. $W \neq \emptyset$ because otherwise $|E| > (24g(g+3)+1)|V|/2 > 6(g-1)+3|V| \geq 6(\gamma(G)-1)+3|V|$, contradicting Equation (1). By Lemmas 3.5 and 3.6, we have that $|B| \geq 3$. We consider two cases: either G is 3-connected or it is not.

G is 3-connected

First we show that $|B| \geq 6$. Suppose $3 \leq |B| \leq 5$. The maximum number of mutually non-inclusive subsets of five elements is $\binom{5}{\lceil \frac{5}{2} \rceil} = 10$, [Bol86, Chapter 3, Theorem 1]. But $|W| \geq 24g(g+3)-4 \geq 92$ and so there must exist two white nodes such that the neighbours of one of them constitute a subset of the neighbours the other. But that contradicts R7 of the reduction procedure.

Let u be a white node of G, and let $v_0, v_1, \ldots, v_{t-1}$ be the sequence of nodes adjacent to u in clockwise order where $t = d_G(u)$. For i from 0 to $t-1$, let F_i be the face containing boundary edges $e_i = uv_i$ and $uv_{i+1 \bmod t}$ in the current graph. If F_i is not a proper 3-face, then add an edge $e'_i = v_i v_{i+1 \bmod t}$ so that e_i, e_{i+1}, e'_i form a proper 3-face.

Let us repeat this operation on all white nodes and denote the resulting embedded graph by G_1. Note that G_1 may contain multiple edges, but that no two multiple edges form the boundary of a 2-cell because G is a connected simple graph and the operation does not create a 2-face. Moreover, no two multiple edges are transformable in $\mathcal{S}_\gamma(G)$ since otherwise the two end nodes of two transformable edges would form a 2-node cut of G which contradicts our assumption that G is 3-connected. G_1 is uniquely determined, up to isomorphism, by the embedding of G.

Now let G_2 be the embedded graph obtained from G_1 by removing all white nodes and their incident edges. Then G_2 is a 2-connected 2-cell embedded graph in which no two multiple edges are transformable.

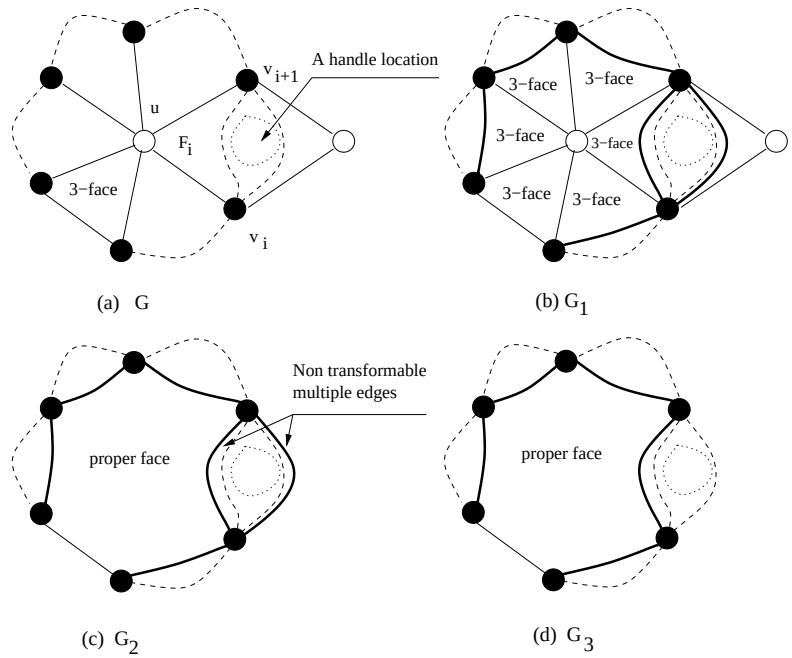

Fig. 3. Embedding Operations

By construction, a white node of G_1 must be located in what becomes a proper face of G_2, and a proper face of G_2 contains at most one such white node in G_1. Since there are at most $d_{G_2}(x)$ faces incident with any node x in G_2, there are at most $d_{G_2}(x)$ white nodes adjacent to x in G_1 and hence

$$d_{G_1}(x) \leq 2d_{G_2}(x) \tag{2}$$

Let G_3 be the graph obtained from G_2 by reducing instances of multiple edges to one edge. See Figure 3 for the construction of G_1 through G_3. Then $\gamma(G_3) \leq \gamma(G) \leq g$ and, by Lemma 3.2, we have

$$|E(G_3)| \leq 6(\gamma(G_3) - 1) + 3|B| \leq 6(\gamma(G) - 1) + 3|B| \leq 6(g - 1) + 3|B|.$$

Since $\delta(G_3)|B| \leq \sum_{u \in V(G_3)} d_{G_3}(u) = 2|E(G_3)|$, $\delta(G_3)|B| \leq 12(g-1) + 6|B|$ and hence $\delta(G_3) \leq 12(g-1)/|B| + 6 \leq 12(g-1)/6 + 6 = 2(g+2)$ and there exists $y \in B$ such that

$$d_{G_3}(y) \leq 2(g+2). \tag{3}$$

Since the G_2 is an embedded graph in a surface of genus $\gamma(G)$ and no two multiple edges are transformable, by Lemma 3.3 there are at most $6g$ multiple edges joining a pair of nodes in G_2. Then by inequality (3) we have

$$d_{G_2}(y) \leq 6g \times d_{G_3}(y) \leq 12g(g+2)$$

By inequality (2) we obtain

$$d_G(y) \leq d_{G_1}(y) \leq 2d_{G_2}(y) \leq 24g(g+2) < 24g(g+3)+1.$$

But this contradicts the assumption we used to begin the proof.

G is not 3-connected

By Lemma 3.4, G has a 2-node cut $\{x,y\}$ such that $G - \{x,y\}$ has a component C such that $G_0 = (V(C)\bigcup\{x,y\}, E(G[V(C)\bigcup\{x,y\}]) \cup \{xy\})$ is 3-connected. Since G contains a path connecting x and y which is internally disjoint with $V(C)$, the embedding of G induces an embedding of G_0 in $\mathcal{S}_{\gamma(G)}$ whether or not xy is an edge of G. Therefore $\gamma(G_0) \leq \gamma(G) \leq g$.

Let $B' = B \cap (V(C)\bigcup\{x,y\})$ and $W' = W \cap (V(C)\bigcup\{x,y\})$. We show that $|B'| \geq 8$. Since W is an independent set and $d_G(v) \geq 3, v \in W$, we must have $B' \setminus \{x,y\} \neq \emptyset$. Let $u \in B' \setminus \{x,y\}$, then $d_{G_0}(u) = d_G(u) \geq 24g(g+3)+1 \geq 97$. Then we have $|B'|+|W'| \geq 98$. If $|B'| \leq 7$, then $|W'| \geq 91$ and $|W' \setminus \{x,y\}| \geq 89$. The neighbours of nodes in $W' \setminus \{x,y\}$ are in B', and so there must exist two white nodes such that the neighbours of one of them constitute a subset of the neighbours the other. This is because the maximum number of mutually non-inclusive subsets of seven elements is $\binom{7}{\lceil \frac{7}{2} \rceil} = 35 < 89$, [Bol86, Chapter 3, Theorem 1]. But this would contradict R7 of the reduction procedure.

Now let $B_0 = B \cup \{x,y\}$ be the black node set for G_0 and consider G_0 to be embedded in $\mathcal{S}_{\gamma(G_0)}$. Let G_1, G_2 and G_3 be derived from G_0 by exactly the same process as described in the previous case. The inequality (2) still holds.

By Lemma 3.2, we have

$$|E(G_3)| \leq 6(\gamma(G_3)-1) + 3|B_0| \leq 6(g-1) + 3|B_0|$$

Since $2|E(G_3)| = \sum_{u \in V(G_3)} d_{G_3}(u) \geq \delta(G_3)|B_0|$ and $|B_0| \geq |B'| \geq 8$, there is at least one black node $z \in B_0 \setminus \{x,y\}$ such that

$$d_{G_3}(z) \leq \frac{12(g-1)+6|B_0|}{|B_0|-2} = \frac{12g}{|B_0|-2} + 6 \leq \frac{12g}{6} + 6 \leq 2(g+3) \qquad (4)$$

Since G_2 is an embedded graph in $\mathcal{S}_{\gamma(G_0)}$ and $\gamma(G_0) \leq g$ and no two multiple edges are transformable, there are at most $6g$ internally disjoint edges joining a pair of points, by Lemma 3.3.

Then, by inequalities (4) and (2), we have

$$d_{G_2}(z) \leq 6g \times d_{G_3}(z) \leq 12g(g+3)$$

Therefore

$$d_G(z) = d_{G_0}(z) \leq d_{G_1}(z) \leq 2d_{G_2}(z) \leq 24g(g+3) < 24g(g+3)+1.$$

But this contradicts the assumption we used to begin the proof. □

4 Conclusions

In Section 2 we showed a time complexity of $O(n^2)$ for the reduction procedure. In Section 3 we showed that in any reduced black and white graphs of bounded genus g there exists at least one node of degree at most $24g(g+3)$. Combining these results yields a time complexity of $O((24g^2+72g+1)^k n^2)$ for the basic algorithm, after the reduction procedure has been incorporated.

It would be interesting to know if the other results known for planar graphs, for example, the existence of a linear kernel and the $O(2^{O(\sqrt{k})}n^c)$ algorithm via tree decomposition, can be extended to graphs of bounded genus.

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