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Implementing the Standards: Let's Focus on the First Four

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Abstract

In this paper we express a concern that there is a treasury of "advanced" mathematical ideas and experiences that are natural and accessible to children in grades K-4, and yet appear nowhere in the standard contents of the mathematics curriculum for these grades, including the NCTM Standards. We argue: (1) that Standards 1-4 (mathematical reasoning, problem solving, communication and making connections) are the most significant of the NCTM Standards and cannot be realized without an expanded mathematics content agenda; (2) that the project of implementing Standards 1-4 is intrinsically connected to the issue of mathematical science popularization, and is well-served by approaches that include manipulative, experiential, and open-ended topics based on *deep mathematics*; (3) that literature and literacy provide useful and powerful metaphors for understanding the important issues in mathematics education reform. In the project of enriching the mathematics content agenda for grades K-4 there is an especially important role for the wealth of concrete topics and mathematical models of modern discrete mathematics and computer science.

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1 Introduction

The Curriculum Standards of the National Council of Teachers of Mathematics [8] can be viewed as an attempt to shift attention in the mathematics curriculum to higher level cognitive issues, and away from the traditional focus on the accumulation of low-level rote computational skills (tasks that increasingly ubiquitous machines do quite well). At all levels of mathematics education, and in many different ways throughout modern culture we see this same general shift to higher cognitive issues and skills. A consensus has rightly emerged that one of the principal goals of mathematics education is mathematical literacy and confidence in mathematical modes of thinking. The purpose of this paper is primarily to discuss the role of mathematical content in achieving this goal.

At every grade level, the following four standards appear at or near the head of the list:

Standard 1: Mathematics as Problem Solving.

Standard 2: Mathematics as Communication.

Standard 3: Mathematics as Reasoning.

Standard 4: Mathematical Connections.

We will call these *the First Four.* No doubt they appear at each grade level because they address directly what it means to *do* mathematics. The items that follow the First Four in the various Standards lists by grade level describe, for the most part, new approaches to old content with a minimal amount of new content. We argue that more needs to be done in terms of content — particularly in grades K–4.

For example, what about the following content possibilities (and we will argue, necessities) in the early grades: proof, infinity, variable, logic, induction, recursion and computational complexity?

And what about the following mathematical experiences?

- The experience of a surprising mathematical truth that contradicts intuition.
- The experience of tangible mathematical unknowns.
- The experience of logical paradox.
- The experience of wrestling with the idea of a limit.
- The experience of mathematical exploration.

In the following we will describe ways in which these and other mathematical experiences and concepts that are typically considered advanced, can engage children in grades K-4 (ages 5–9), and why they should be introduced to this age group. Many of the topics by which these ideas and experiences are conveyed are relatively new as mathematics — many are part of computer science and its discrete mathematical roots.

The main points of our argument are summarized as follows:

- The First Four curriculum standards cannot be meaningfully implemented except in the context of a significantly enriched mathematics content agenda. They are not independent of content issues.
- There is natural compatibility between the First Four curriculum standards and the goals and methods of effective science popularization.
- Literature and literacy provide useful metaphors for understanding many of the important issues in mathematics education.
- The project of reforming science education in the elementary grades is likely to benefit significantly from an enrichment of the mathematics agenda.
- Discrete mathematics and computer science have an important role to play as sources of content enrichment for the elementary grades.

We hypothesize that all of the problems with mathematics education at all levels are abundantly represented in the first five years of school, and for that reason, draw our comments from our experiences with children in classrooms at these grade levels.

By the end of even the first year many, if not most, of the children we have met have already formed a dismal impression of mathematics, considering it a boring and intimidating discipline devoted primarily to speedy and accurate manipulations of numbers. By the end of the fourth year they have typically had an abundance of the traditional experiences of school mathematics: the meaningless seat work, the rote memorization of procedures, the stilted word problems and pointless obscure vocabulary, the anxiety of the parents and teachers, and the testing that separates the winners from the losers. They have already experienced "mathematics as crowd control"[1] where the reward for mastering a drill sheet is — another drill sheet.

One of the ironies of this fundamental age group is that their playground culture is rich with combinatorial games, with riddles and word-play, with informal discussions of infinity, space-time, and the Liar Paradox. They are busy with topological and dynamic amusements such as tether ball, jump rope, cat's cradle and braids. These activities and puzzlements are

[1] See *the paranoid theory of mathematics education* in [5].

in many ways closer to the spirit of mathematics as it known by mathematicians than what is presented as "mathematics" in the classroom.

2 The First Four: What do they really mean?

Mathematicians understand that making connections, communication, problem-solving and reasoning are at the heart of their discipline. As measurable skills, however, these are nebulous — far more difficult to teach and track than the ability to count, compare, or compute. One of the things that is commonly happening in practice as school districts and curriculum developers wrestle with the Standards is that the First Four are in many cases being split off and treated differently from the rest. In particular, they are in many cases being interpreted merely as process standards having no particular connection to any kind of mathematical content.

One well-meaning principal of an elementary school in British Columbia, which has been used as a model for curriculum reform, put it this way. "These four standards are really important — we handle them elsewhere in the curriculum!" By this was meant that communication skills are practiced in creative writing, problem-solving skills are practiced in designing art projects, etc.

Our central argument here is that the First Four cannot be realized without an expanded agenda of interesting mathematics and mathematical experiences to reason, communicate and problem-solve about. We simply cannot realize these standards by means of classroom discussions about our ideas for doing long division or naming triangles. If the current impoverished K-4 mathematics agenda is not capable of supporting any meaningful realization of the First Four, we must look to all of mathematics for expanding the range of ideas that are brought to the K–4 classroom.

We believe the K–4 content curriculum should include anything and everything suitable for a Mathematical Sciences Museum and thus the project of realizing the First Four for K-4 is naturally allied with the vital project of mathematics popularization for all ages. In these first years, an enduring sense should be formed of what mathematical science is about and how it feels to participate in this adventure of the human spirit, central as it is to all of modern science and technology. [2]

In fact, science popularization is inherently concerned with the K-4 audience because science museum exhibits are, more or less, designed for the 4th grade audience in order to

[2]Notice that if in this sentence the words "mathematical science" are replaced by "print literacy," then the result is a common-place. Not only do children routinely master the decoding of print in K-4, but they engage exciting poetry and stories (including their own) and form a basic sense of why one would want to read and write.

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Figure 1: A map which will require 3 colors to be colored correctly.

be just about right for children, grandparents and everyone else in between.

For scientists and mathematicians, the K-4 audience is a delight. They are full of vibrant curiosity and enthusiasm. They are endowed with natural tendencies to abstract representation and the play of ideas. (Think of the odd bits of wood they have asked you to regard as a space laser.) In the next section, we describe some of the "advanced" mathematical ideas that can be engaged by this age group.

3 Some Content Examples for the First Four

The purpose of this section is to describe some examples of how advanced mathematical ideas can be engaged by children in grades K-4. One of the most fundamental appreciations that one can have of mathematics is a sense of the power, the sheer variety and the marvelous interconnections of mathematical models of things in the world. We may begin with the K–4 audience by exhibiting and engaging a rich collection of examples of mathematical models.

3.1 Example 1: Map and Graph Coloring

The basic **Map Coloring Problem** is that of trying to discover the minimum number of colors needed to properly color a map. A map is properly colored if no two countries sharing a border have the same color. (See Figure 1.)

This problem (like any of hundreds of such combinatorial optimization problems) can be presented in a classroom setting by doing the following. [3]

1. Beforehand, make up and photocopy 3 or 4 maps of varying sizes, such as with 5, 10 and 20 regions. (Do not "solve" them.)

2. In class, discuss how maps are ordinarily colored, how regions on the map that share a boundary are colored different colors so that they are not easily confused. Discuss also, how it would make sense commercially to color maps with as few colors as possible, due to the cost of ink, the complexity of printing many colors, etc.

[3] Detailed instructions for classroom use of this and other problems along with sample handouts, ideas for discussion, and explanations of their relationship to the whole of mathematics can be found in [3].

3. Pass out the maps and invite students to find ways to color them with as few colors as possible, working individually or in groups, as they prefer.

4. Be an attentive listener and facilitator. Encourage the children to describe their ideas for solving this problem and to explain what they are doing to each other.

5. Afterwards, have the children write about their ideas, draw maps of their own to color, and/or share the activity with a different group of children.

The coloring problem is one of the great gems of discrete mathematical modeling. Some of its applications include: the assignment of non-interfering frequencies to radio stations, the timing of traffic lights, the scheduling of meetings and machines, and the scheduling of garbage truck routes. Coloring is also an activity to which the K-4 audience is already natively inclined. There is some satisfaction in connecting this ordinary childhood artistic activity to the deep and important mathematics that concerns it. Surprising to most people is the fact that coloring problems (of various kinds) remain a subject of vigorous mathematical investigation. They are important to all kinds of discrete mathematical modeling, including, for example, the analysis of DNA sequences.

3.2 Coloring: What's In It?

In K–4 classrooms, where children are puzzling over finding the minimum number of colors for various maps, all kinds of interesting and deep mathematical issues naturally arise. We are concerned that if these ideas are left off of the content agenda, teachers will lack an adequate reference framework to appreciate, stimulate and support the problem-solving strategies that the children will invent. The following is an unsystematic inventory of various fragments of our classroom experiences with the coloring problem, pointing to various "advanced" mathematics content that emerged.

3.2.1 "Two is not enough!"

What typically happens with a hypothetical map M with chromatic number 3 (Figure 1) is that someone first colors it with 7 colors, and then someone colors M with 5 colors, ... the number gradually improves. But eventually we are left wondering (publicly, as we celebrate this progress) whether we can do it with 2 colors. Inevitably, some child will figure out that (and explain energetically why) two is not enough for M, typically by finding three regions each of which borders the other two. The moment when a child gives that excited shout needs to be appreciated as a "teachable moment" for the fundamental topic of **mathematical proof**. A teacher not equipped with the idea of the importance of mathematical proof, and twocol.eps

Figure 2: Maps drawn as closed curves can always be colored with two colors.

expecting to encounter and develop this concept, is not equipped to fully appreciate and empower the problem-solving going on.

3.2.2 "I did it with two!"

Consider the same scenario with a different map M' having chromatic number 2 (Figure 2.). We have the same gradual improvements. First there is a solution with 5 colors, then one with 4 or 3 colors. Finally someone shouts that they have done it with 2 colors.

If we interview the children who have found a 2-coloring for M', asking about their method and their ideas, we usually find that they have hit upon the following systematic approach. They first color a region, with, say, red. And then choose another color, say, blue for those neighboring regions that are forced to be blue because they share a border with the first region. And then (conservatively) they proceed by coloring red those further regions that share a border with the newly-colored blue ones, and thus are forced to be red, and so on.

This is a very interesting strategy! In fact, it is an **algorithm** that can serve to determine for any map whether two colors is enough, and do so with great **algorithmic efficiency/complexity**. It can be compared to a different (but also interesting and respectable) **greedy algorithm** that many other students will discover: pick up a crayon and (at random) use it to color regions until it can't be used anymore, then pick up a new crayon and repeat the process until all regions are colored. A teacher (and a curriculum agenda) not equipped with the idea of an **algorithm** is not equipped to appreciate the problem-solving going on here, the ideas that are emerging, and their substantial ultimate significance in mathematics education.

Here is also an opportunity to point out to the children one of the most important **unsolved problems** in all of mathematics and computer science. This is that while we have the efficient algorithm for 2-coloring sketched above, no one knows whether there is a fast way to find out whether 3 colors are enough. It seems a good thing not only to share with children significant problem-solving situations that have no single right answer, but also situations where no one, not even the adults, presently knows the answer.

3.2.3 "I want to try to do it with 3!"

When the children are working with crayons, the natural thing to do at first is to dive in and color as well as you can. However, once students want to experiment so as to truly minimize their colorings, a certain weakness in using crayons becomes apparent — it is impossible to back up! Once a region has been colored red, it's messy (if not impossible) to try to change it to green. On their own, or with minimal encouragement, some children will switch to using colored tokens to mark the colors that they assign to the regions. This provides a far more powerful means to try to achieve an optimal coloring.

In one classroom, a teacher observing the children moving the colored markers around on the maps remarked, "That's a higher level of abstraction." The teacher obviously (and rightly) felt the need to appreciate this more powerful problem-solving approach in some way. Rather than rely on psychological concepts for this, we can appreciate what's going on in a straightforward mathematical way: the regions are now functioning (manipulatively) as **variables** that can be conveniently instantiated to a color value by a marker. This is precisely why this is such a powerful problem-solving strategy, and a good demonstration of why the concept of a variable is so fundamental in mathematics. If **variable** is not on the content agenda, then teachers are left to ad hoc psychological appreciations, with no sound connection to the enduring and important mathematical ideas that are emerging in the children's activity.

3.2.4 "These maps can always be done with 2!"

If you place your pen on a piece of paper and draw any sort of intersecting continuous curve, eventually returning your pen to its starting point without lifting it from the paper, you will have drawn a map that is 2-colorable! (See figure 2.) Try it out. Colored, it looks like the kind of "psychedelic checkerboard" that Salvador Dali might have preferred. It is generally regarded as somewhat surprising that these kinds of maps are always 2-colorable. How can we be convinced about that this is true?

One way to explore being convinced is to make a loop of string. Imagine that it is black string, imitating the black ink of a pen that would draw such a map. Surely you will agree that you could lay the loop of string right on top of the curve that you drew. If it were just the string, lying like that on the white paper, we might think, "What a mess!" We might decide to gradually, very slowly, one step at a time, move the loops apart. We might in this way obtain a very boring situation: the string is now just lying in a loop that does not intersect itself. If this were a map, it would just be one island and the sea surrounding it — of course we have no trouble 2-coloring this!

Now let's slowly go backwards, gradually putting things back the way they were. At graphcol.eps

Figure 3: This graph is colored correctly with 3 colors.

each step of the way we will notice that we make one of a few kinds of moves, and in each case, the property of the map being 2-colorable is preserved! Now we see why all these maps are 2-colorable: they are all just mixed-up forms of the One Two-Colorable Island (and the mixing up doesn't hurt anything). There is a lot of fun and contemplation in this for young children. It is really the essence of a proof by **induction** of this surprising theorem. (No matter how old you are, you really should try this out with a piece of string and two kinds of colored markers, and see that induction is, after all, really quite suitable for 7 year-olds.)

3.2.5 "This one can be done with 3! See if you can find how!"

A problem closely related to map coloring is graph coloring. A graph is a network of dots (called vertices) connected by lines (called edges). The vertices of a graph are properly colored when no two vertices joined by an edge receive the same color. (See Figure 3.)

Several children in one of our second-grade encounters spontaneously decided that the 3-coloring puzzles for graphs that were passed out were so much fun, they would have **Graph 3-Coloring** as an activity at their birthday parties!

Here is a little mystery for further exploration. How was it possible to announce to them, "It is easy to draw a graph that can be colored with 3 colors so that you know exactly how to color it, but other people will have a hard time figuring out how." The puzzle is solved with a very entertaining activity: begin by making a polka-dot pattern of 3 kinds of colored dots. On top of this lay a fresh piece of paper, and tracing through, make a circle around each dot. Now add edges between these circles, but only between circles that surround differently colored dots! In this way you have created a graph for which you know a secret 3-coloring, but it might be pretty hard for someone else to find one. This is a kind of combinatorial **one-way function**, a topic of profound importance in modern mathematical cryptography (for further explorations beginning from this point and involving **polynomials** as encryptions of public-key messages see [6]).

4 Other topics of interest for the early grades

We next describe (a bit more telegraphically)a few more topics that have proved fruitful in exploring the first four standards in the early grades.

muddy.eps

Figure 4: A map that can be used for the Muddy City Problem.

lhtref.eps

Figure 5: A left-handed trefoil knot.

4.1 Minimum Weight Spanning Trees

We have come to call this the "Muddy City Problem". (See figure 4.) The scenario is a city with unpaved roads in which transportation becomes impossible when it rains. The vertices of the graph represent houses and the edges are roads. The labels on the edges of the graph are the costs of paving each segment of road. The question becomes: What is the least expensive way to pave roads so that everyone can get to everyone else's house when it rains (even if it is by a circuitous route)?

In the attempts to find an optimal solution for a given weighted graph, a typical classroom experience invokes a hurricane of arithmetic as children work to create ever better solutions, and to match the best that have been found so far. The are many interesting nontrivial ideas and observations that children will typically make and be prepared to explain and argue: such as the fact that an optimal solution has no cycles. Here again we have the vital content of **mathematical proof** arising. The fact that there is a (surprising and elegant) fast algorithm for the MST problem raises the issues of **algorithm** and of **algorithmic efficiency**.

Suppose we notice that all the best solutions for a given graph (optimal solutions are generally not unique), all involve the same number of edges. One will easily note that we have here again an opportunity for a simple, visually presented argument by **induction** to answer this.

4.2 Knot Theory.

The Canadian Navy donated to us a number of large ropes, and we have had wonderful experiences presenting some of the rudiments of knot theory in elementary classrooms. This is an excellent topic for mathematics popularization for several reasons. First of all, it is first-rate mathematics that has recently moved center-stage in the research world in a very exciting way. Secondly, everyone uses knots, and almost no one is aware that they are an object of mathematical investigation. To share the fact that there is a *mathematics* of knots is a powerful illustration of the richness of mathematical science. Finally, knot theory is enormously open to manipulative presentation.

One can ask about mirror-image knots. Is the left-hand trefoil the same as the right hand one? On some occasions we have brought along a large portable mirror to show that the one is indeed the mirror image of the other. Knots (once oriented) support a well-defined notion of (abelian) "multiplication" (having even a prime factorization theorem) that is open to engaging manipulative exploration. Here we have **symmetry** and **mathematical operations**.

There are many more such mathematical topics supporting rich opportunities to realize the First Four. The main point is that in really engaging these or any other opportunities for mathematical problem-solving and communication worthy of the name, "advanced" mathematical ideas will naturally and inevitably arise and should be both expected and deepened as much as possible.

5 Parallels Between Mathematics and Literature Teaching

We have found the analogies between print literacy and mathematical literacy to be both strong and productive for generating ideas and methods for improving education in mathematics during the critical, formative elementary school years. Lacking a scapegoat and deterrent to risk-taking as formidable as The Debacle of the New Math, elementary school language arts educators have benefited from 25 years of experimentation and critical evaluation leading to teaching methodologies and classroom structures aimed less at making the student a skilled automaton with the structure of written language, and more focused on the development of the student as a literate person.

It is no less difficult to define what a literate person is than it is to describe what it means to do mathematics. For example, it is not sufficient to say that someone is literate because they know a lot of words, read fast, spell and punctuate Standard English accurately, speak several languages, or can pass tests about all of the books on a certain prescribed list. Yet so-called literate people can do many or all of these things. Likewise, in mathematics, developing a straight-forward definition of literacy is no less complicated or controversial.

Mathematics and literature have much in common. The construction, examination and communication of ideas is central to both disciplines. In each discipline these activities are carried out within forms. These forms are often misconstrued to be the discipline itself. Each discipline is so vast, with such a rich and long tradition, that no individual can claim to grasp it in its entirety, yet any aspect is accessible to the dedicated participant. In both mathematics and literature, the participants in the discipline form a community in which innovations and content are shared and examined. The most renowned and influential participants in the community achieve their position after a long period of initiation and experience, much of which, in the early years, occurs in schools.

Elementary school language arts teachers ask the following questions when they plan and evaluate their lessons [1]:

- How can we prepare students to become creative participants in a community where the formulation and communication of ideas is fundamental?
- How can we, with materials and tools that are on hand now, teach them to appreciate the vast and ever-changing quantity of material they will encounter in their lifetimes, to assimilate new things, and develop taste as they mature intellectually?
- What must we do so that all students acquire the complex, interrelated skills necessary to do all this?

Similar questions should be fundamental to mathematics teaching in the formative years.

The following insights borrowed from language arts teachers' examination of their goals and methodologies over the last 3 decades are most useful for considering the direction that change in mathematics education should take [4].

1. Children benefit from exposure to a rich variety of content without regard for hierarchical sequencing of material. Statements of "developmental appropriateness" must be taken in a large context.

2. Students are drawn forward by exposure to material that they can understand but which is beyond their capacities to produce.

3. Although skills matter, experience in the discipline cannot be secondary to mastery of them; teachers must find ways to monitor and nurture skill development within the context of meaningful and stimulating (self-selected) projects.

4. Students must be steered towards mature and independent self-selection of content materials and individual/small group projects which they undertake.

5. Peer communication about their ideas is not only critical, but inevitable. Teachers must learn to exploit, not suppress the classroom culture.

6. Students must be given large blocks of time to read, think, talk to one another, share, argue, and write down their ideas. The classroom should be a microcosm of the community into which the students are being initiated.

7. The teacher is neither spectator nor ambassador from the community into which the students are being initiated, but a participant and a practitioner.

These questions and insights are less about language teaching than about teaching in general. They are representative of a largely grass-roots movement in language teaching reform which came to be termed **Whole Language**.

The Whole Language connection [2] has proved to us to be an enormously useful handle in speaking to experienced elementary school teachers about mathematics education reform. [4] Many elementary school teachers have spent many years wrestling with these issues. What they typically sorely lack is any sense that mathematics *has* a literature, that it supports any kind of thinking or activity remotely resembling literacy. We can help teachers and parents appreciate and understand reform in mathematics education by appealing to their understanding and experience with print literacy.

In all kinds of contexts, the literacy connection has proved useful. Here are a few examples of what we have come term "standard conversations" with parents and teachers, and how they can be answered by looking through the lens of literature.

"Why does my child need to know about coloring or knot theory?"

Does your child need to read *Charlotte's Web* or *Huckleberry Finn*? Does your child need to know about dinosaurs or outer space? (It is sad that mathematics is so universally associated with such a miserliness of spirit.)

"It's important to teach arithmetic. So now you are saying that it is important to teach coloring as well?"

It's important to teach spelling, but it's also important to read and enjoy books. What particular books these are is not so important, but they should be rich and interesting stories. It is much the same with mathematics.

6 What Is To Be Done?

The elementary school principal who said, "These four Standards are really important — we handle them elsewhere in the curriculum!" was not (as one might first suspect) making an easy mistake. At this particular school, meaningful contexts for learning and the development of communications skills are highly valued. An intelligent and demanding (and yes, it includes phonics) Whole Language approach to print literacy is a deeply-rooted practice at this "charter" school which has served for years as an important model for curriculum

[4] Current controversies regarding forms of Whole Language have done nothing except strengthen this connection, as they bring out an important and inevitable tension between skills development, and issues of motivation, participation and meaningful context. These controversies serve as a useful warning about trivializing literacy education of any kind.

innovation in British Columbia. The traditional mathematics content agenda at the elementary grade levels, however, simply does not provide adequate opportunities to realize the obviously important First Four *in mathematics*, so in order to address them, teachers must turn to opportunities elsewhere in the curriculum.

There are several things that we in the Discrete Mathematics and Computer Science communities need to do:

- We need to pay far more attention to the needs and opportunities in the early and formative years of schooling.
- We need to get the message out to the elementary schools that integrating the intellectual core of computer science (and its roots in discrete mathematics) into the curriculum is of *far* greater importance than worshiping in the expensive Cargo Cult of computers-in-the-classroom. (For further discussion of this point see [6].)
- We need to make the connection between mathematics education and mathematics popularization. In the areas of discrete mathematics and computer science we have enormous resources of important, accessible mathematics for this purpose.
- We need to establish connections between mathematics education and literacy education, especially at the K-4 level. Such connections are likely to significantly strengthen *both* educational agendas. The communication of mathematical thinking and argument, and the formulation mathematical models and conjectures constitute challenging important kinds of writing tasks. We need to encourage the development of whimsical, lengthy, content-rich children's mathematical literature. We need story problems that are real stories, not "Farmer Brown wants to build a rectangular fence... ." For example, we need 30-page stories with characters, pictures, maps, and dialogue that incorporate interactive problem-solving. We need to be training mathematics/cross-disciplinary students (perhaps educational computer games designers) at the universities to *create* this kind of literature.
- We need to support teacher professionalism, and serve as (energy-efficient) catalysts for change, by organizing and involving mathematical science undergraduate and graduate students in outreach from the universities (perhaps as a component of service education programs). We need to similarly organize summer in-service institutes for teachers, and mathematical science summer camps for kids.

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