Fixed-Parameter Tractability, Relative Kernelization and the Effectivization of Structural Connections

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Abstract.

The central concept of parameterized complexity is the generalization of P, *fixed parameter tractability* (FPT), solvability in time $f(k)n^c$ where n is the input size, k is the parameter, c is a constant, and f is an arbitrary function. Another view is based on the *Kernel Lemma:* A parameterized problem is FPT if and only if there is a P-time algorithm to transform (x, k) into an equivalent *kernel* (x', k'), where: (1) $k' \leq k$, (2) $|x'| \leq g(k)$, and (3) (x, k) is a yes-instance iff (x', k') is a yes-instance. In recent years there has been an explosion of research in FPT kernelization, because this view of FPT allows the systematic investigation of pre-processing, something that is difficult to formulate in the classical framework. Such investigations have revealed that the subject has unexpected mathematical depth and strong practical applications. We investigate a generalization.

We initiate a systematic study of *relative kernelization* and the algorithmic effectivization of structural connections. Letting $\Gamma(G)$ denote the maximum length of a cycle in G, and $tw(G)$ the treewidth of G, then we have the *connection*: $\Gamma(G) \leq k$ implies $tw(G) \leq k$. A effectivization is given by the "win/win" algorithm of Fellows and Langston that in P-time either determines that $\Gamma(G) > k$ or produces a tree decomposition of width at most k. Win/win's play an important role in many FPT algorithms. Mathematics offers many such existential (extremal) connections between various parameters of objects such as graphs. We initiate the study of complexity issues of effectivization. This leads to multivariate generalizations of FPT by varying (2), replacing the bound on the size of x' by a bound on $\mu(x')$ for some other measure μ (for example, treewidth). We explore relative kernelization and its limits.